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THE RELATION BETWEEN BUBBLE FREQUENCY AND DIAMETER DURING NUCLEATE POOL BOILING

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Abstract—New data concerning the frequency and diameter with which bubbles leave a given bubble site in nucleate pool boiling is presented. The new data was obtained from a boiling study of liquid nitrogen. It is shown that the existing correlation relating the frequency and diameter does not predict the new data. From a dimensional analysis coupled with available experimental data a new relationship is developed, viz. $f \,.\, D^{1/2} = \text{const.}$ A simple force balance on the departing bubble is shown to indicate the same result.

INTRODUCTION

ONE of the least understood and most important aspects of heat transfer at present is the boiling process. Today, because of the increasing desire to transfer more energy per unit area of heat exchange surface, one naturally turns to boiling as a means of accomplishment. As evidenced by the number of attempts, it has been exceedingly difficult to find a heat transfer correlation that will satisfactorily predict boiling heat transfer under a variety of conditions, mainly because there are so many variables which enter the boiling process.

One of the variables important in many pool boiling heat transfer correlations is the product $f \,.\, D$, where

f = the frequency with which the bubbles leave a given site, and

D = the diameter of these bubbles at the instant they leave the surface.

An example of how the product enters a heat transfer correlation is given by the correlation proposed by Rohsenow [1], viz.

$$q'' = c\, h_{fg}\, D^3 f\, \rho_v\, n \tag{1}$$

where

q'' = the heat transfer per unit area of surface,

h_{fg} = enthalpy of evaporation,

ρ_v = vapor density,

n = number of bubble sites per unit area,

and c = is a proportionality constant.

It was observed by Jakob [2] that the bubbles leave a given bubble site at roughly equal time intervals, and further that the bubbles from this site are nominally of equal diameter. Jakob presented some data and noted that over a range of diameters the data could be approximated by the expression

$$f \,.\, D = \text{const.} \tag{2}$$

Zuber [3] has proposed an equation describing the product of frequency and diameter in terms of fluid properties. He noted that the velocity of rise of bubbles in a liquid can be expressed as†

$$u_\infty = 1{\cdot}18 \left[\frac{\sigma g \Delta\rho}{\rho_L^2}\right]^{1/4} \tag{3}$$

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† It should be noted that this expression is only valid over a certain range of Reynolds numbers. See for example Grassmann [4].

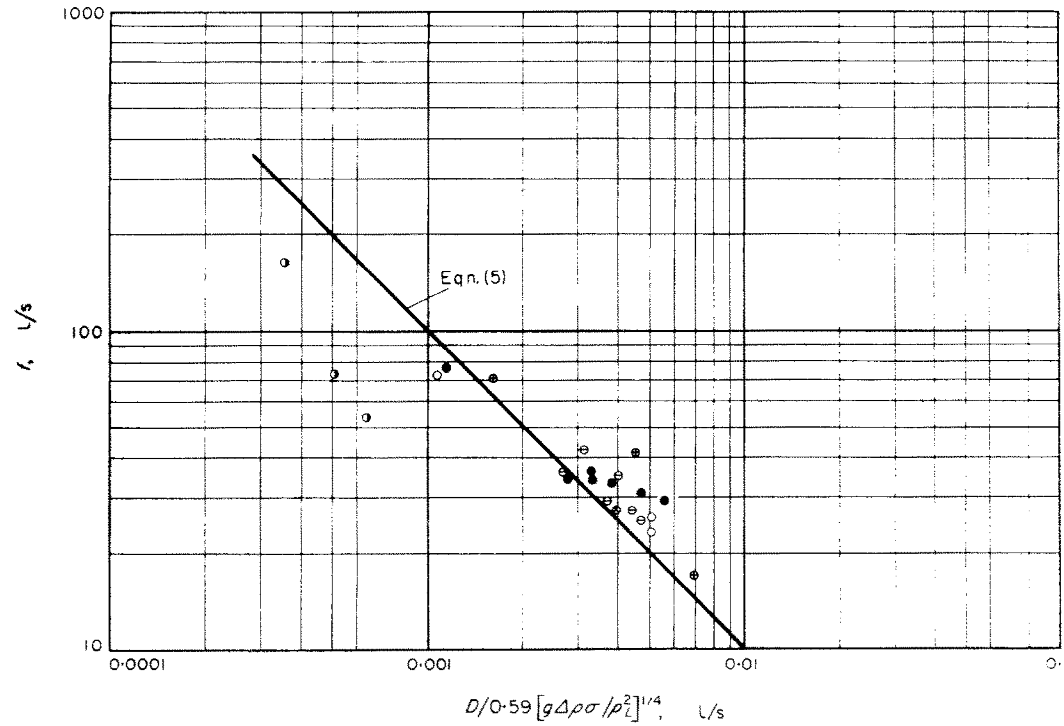

FIG. 1. Frequency of vapor bubble formation as a function of fluid properties and bubble diameter for:
●, ○ Water, Fritz and Ende [6];
⊖ Water, Yamagata and Nishikawa [7];
⊕ Carbon Tetrachloride, Jakob and Linke [8];
⊗ Methylalcohol, Westwater and Perkins [9]; and
◑ Nitrogen.

where

u_∞ = bubble rise velocity,
σ = surface tension between liquid and vapor,
$\Delta\rho$ = the difference in density between the liquid and vapor phases,
ρ_L = liquid density,
g = acceleration of gravity,

as shown by Peebles and Garber [5]. He then assumed that

$$f \, . \, D \propto u_\infty \quad (4)$$

and found

$$f \, . \, D = 0{\cdot}59 \left[\frac{\sigma g \Delta\rho}{\rho_L^2}\right]^{1/4}. \quad (5)$$

This expression fits the data as shown in Fig. 1 satisfactorily over a limited range of bubble diameters.

RESULTS

It was felt, however, that data over a wider range of frequencies and diameters was needed, and therefore the present work was undertaken. Data was taken for nucleate boiling in liquid nitrogen. High-speed motion pictures were taken of the bubbles during boiling, and the results are shown in Table 1. These results represent the arithmetic average values of the

Table 1

f (1/s)	D (cm)
165	0·021
72	0·030
54	0·038

frequency and diameter (at breakoff) at each of three locations. If the results are reduced and plotted as shown in Fig. 1, it is seen that the nitrogen data falls well below the frequency prediction of equation (5). Stephan [10], working with boiling in liquid freon, has observed the same qualitative results.

ANALYSIS

A dimensional analysis, assuming that $f \cdot D$ is a function of D, σ, ρ_L, and $\Delta\rho$, shows that

$$\frac{f \cdot D^2 \rho_L^{1/2}}{(\sigma D)^{1/2}} \propto \left(\frac{g\Delta\rho D^2}{\sigma}\right)^a$$

is a possible relationship describing frequency and diameter in terms of liquid and vapor properties. It is easily shown that the left side of the above proportionality represents the square root of the ratio of inertia to surface forces, and the right side represents the a^{th} power of the ratio of buoyant to surface forces. If $a = \frac{1}{4}$, the result of Zuber is obtained.

The plot of Fig. 2 shows, however, that $a \approx \frac{1}{2}$ fits the experimental data satisfactorily. This means that

$$f \cdot D^{1/2} \approx 0{\cdot}56 \left(\frac{g\Delta\rho}{\rho_L}\right)^{1/2} \qquad (6)$$

for a given fluid. Also, since (at least far from the critical point)

$$\Delta\rho \approx \rho_L$$

$$f \cdot D^{1/2} \approx 0{\cdot}56(g)^{1/2} = 17{\cdot}5 \text{ cm}^{1/2}/\text{s} \qquad (7)$$

with no dependence on fluid or vapor properties. This is the case for the available data as shown in Fig. 3.

This relation is not too surprising if one considers the following. A bubble at the instant of breakoff is acted upon by various forces. There is a force between the bubble and the solid surface. Liquid flows over the bubble in various erratic and unpredictable ways, pushing it and pulling it. But the predominant force (at least for low heat flux where bubble columns do not interfere with one another) is due to gravity.

For a bubble at the instant of breakoff then

$$F = g\Delta\rho V = \frac{\mathrm{d}}{\mathrm{d}t}(mv) \qquad (8)$$

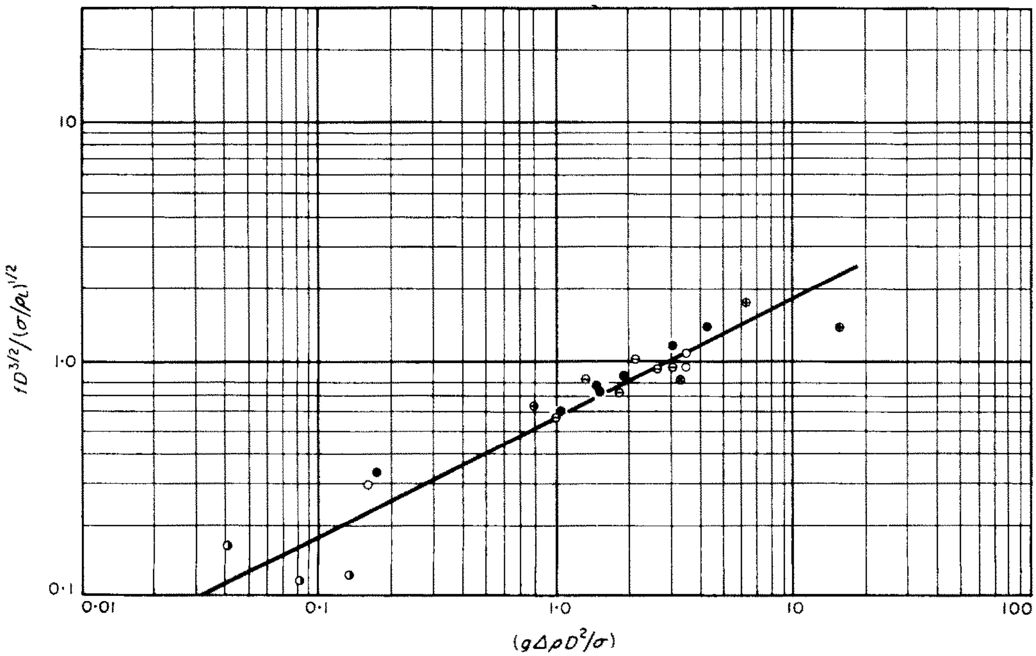

FIG. 2. Non-dimensional formulation of the frequency–bubble diameter relationship.

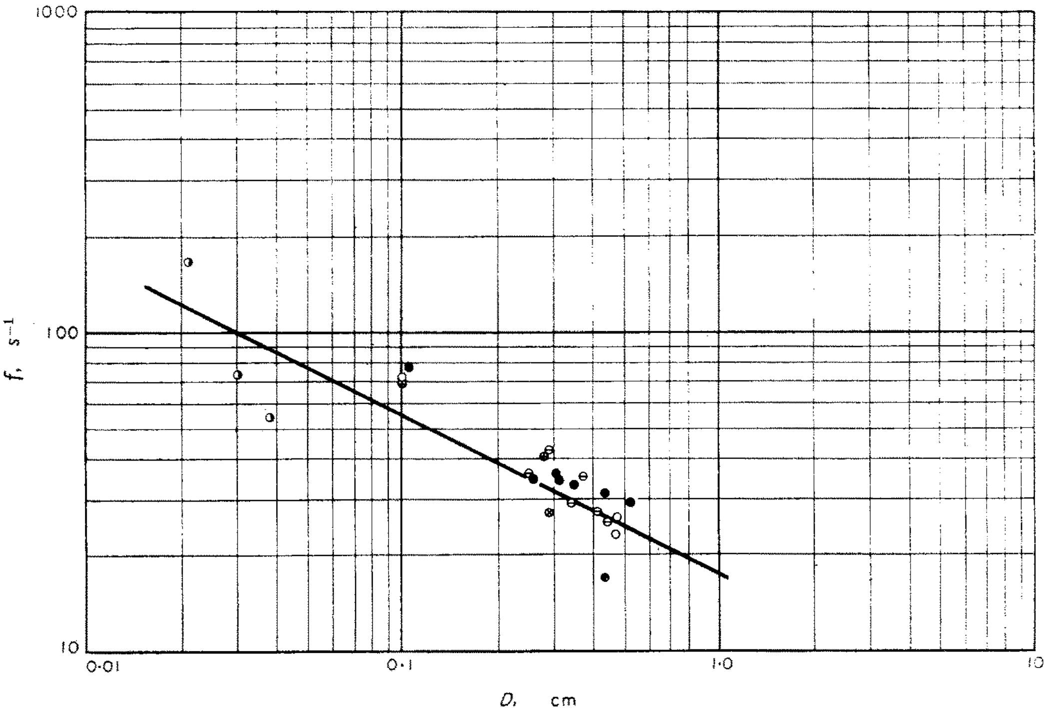

FIG. 3. Frequency of vapor bubble formation.

where

F = the predominant force acting on the bubble,
V = bubble volume,
t = time,
m = accelerating mass,
v = velocity.

Since the bubble is not growing much at this time (see e.g., Jakob [2]),

$$F \approx m \frac{\mathrm{d}v}{\mathrm{d}t}. \tag{9}$$

Further, if we assume that the inertia retarding the bubble is due to the liquid, then

$$\frac{\mathrm{d}v}{\mathrm{d}t} \approx g \frac{\Delta\rho}{\rho_L} \approx g. \tag{10}$$

Finally, taking

$$f^2 \, . \, D \propto \frac{\mathrm{d}v}{\mathrm{d}t} \tag{11}$$

it is seen that

$$f \, . \, D^{1/2} \propto g^{1/2} \tag{12}$$

as was obtained experimentally.

CONCLUSIONS

It seems that equation (5) is not a general relationship. While more measurements, especially at diameters outside the presented range, are needed, equation (7) offers an interesting possibility for predicting a frequency spectrum given a bubble diameter spectrum.

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Résumé—On présente ici de nouveaux résultats relatifs à la fréquence et au diamètre avec lesquels les bulles quittent leur site de formation dans la masse en ébullition nucléée. Ces nouvelles données ont été obtenues pour l'étude de l'ébullition nucléée de l'azote liquide. On montre que la corrélation qui existe entre le diamètre et la fréquence des bulles n'est pas en accord avec les nouveaux résultats. Une analyse dimensionnelle associée aux données expérimentales valables a permis d'établir une nouvelle relation $f \, . \, D^{1/2} =$ const. Un simple équilibre des forces sur la bulle qui s'échappe conduit au même résultat.

Zusammenfassung—Aus einer Untersuchung des Blasensiedens von flüssigem Stickstoff bei freier Konvektion werden neue Werte von Frequenz und Durchmesser beim Abreissen von Dampfblasen an Siedekeimen mitgeteilt. Während die vorhandenen Beziehungen die neuen Messwerte nicht bestätigen, lässt sich aus einer Dimensionsanalyse die Beziehung $f \, . \, D^{1/2} =$ const. ableiten. Eine einfache Kraftbilanz an der abreissenden Blase führt zum gleichen Ergebnis.

Аннотация—В статье приводятся новые данные о частоте отрыва пузырьков пара, диаметре их при отрыве от места образования при пузырьковом кипении. Новые данные получены на основе изучения кипения жидкого азота. Показано, что существующая корреляция, связывающая частоту и диаметр, не соответствует новым данным. С помощью анализа размерностей и имеющихся экспериментальных данных выведено новое соотношение: $f \, . \, D^{1/2} =$ const. Показано, что уравнение равновесия сил, действующих на отрывающийся пузырёк, приводит к тому же результату.